EE105
Microelectronic Devices and Circuits:
MOS Small-Signal Model

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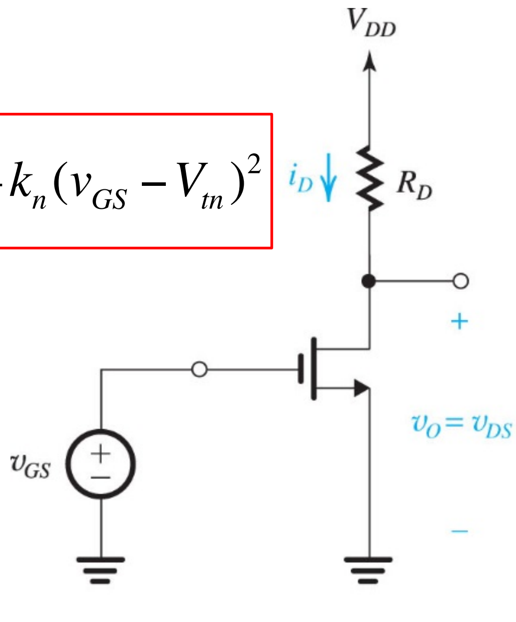
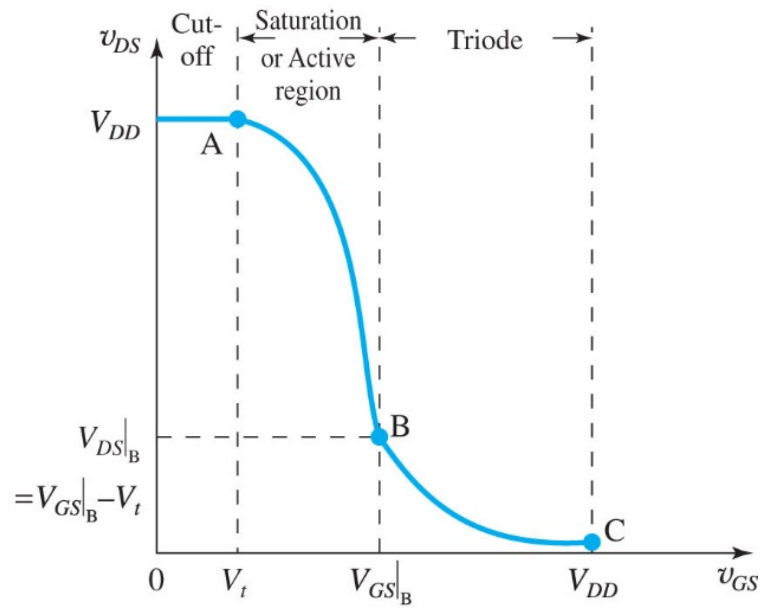
Voltage Transfer Curves

$$i_D = \frac{1}{2}k_n(v_{GS} - V_{tn})^2$$



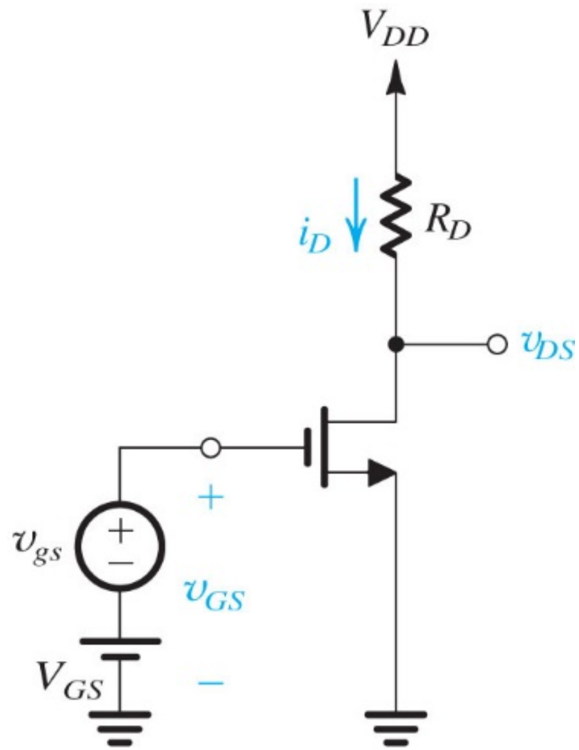
(a)

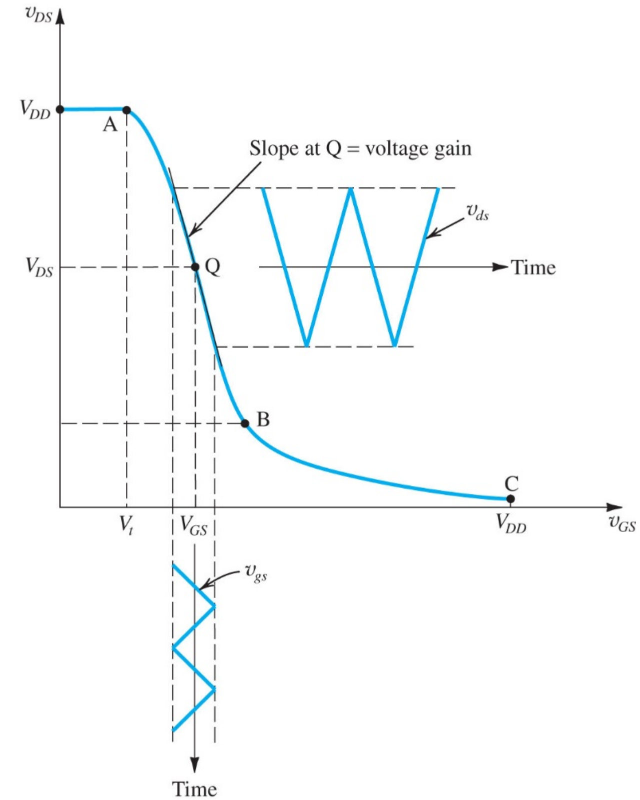


(b)

- **Transistors are transimpedance amplifiers**
  - **MOSFET: v_{GS} controls i_D**
- **Adding load resistors converts current to voltage → voltage amplifiers**
  - **Drain voltage free to change in saturation mode**

Voltage Amplifier





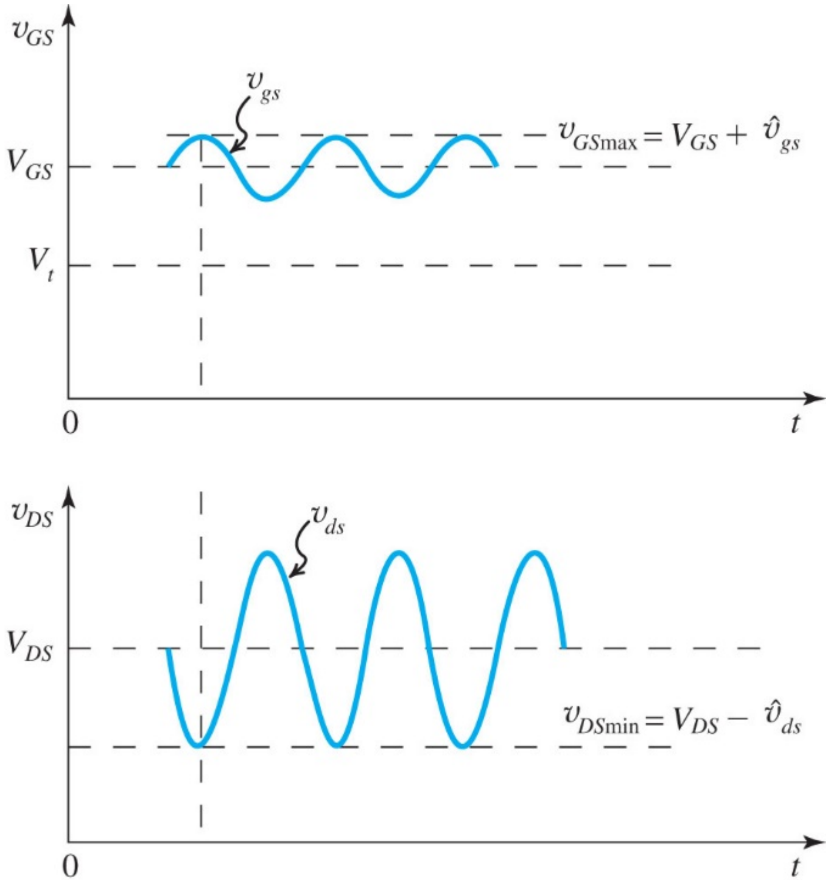
- **DC bias at Q point (Quiescent point)**
- **Small-signal input superimposed on DC bias voltage**

$$v_{DS} = V_{DD} - \frac{1}{2} k_n \left(v_{GS} - V_t \right)^2 R_D$$

$$A_V = \left. \frac{dv_{DS}}{dv_{GS}} \right|_Q = -k_n \left(v_{GS} - V_t \right) R_D$$

Voltage Gain: $A_V = -k_n V_{OV} R_D$

Maximum Input Signal Amplitude



To keep the MOSFTET in Saturation region (linear part of I-V curve):

$v_{DS} \geq v_{OV} = v_{GS} - V_t$ at all time.

This condition is met if

$$v_{DS\min} = V_{DS} - v_{ds} \geq v_{GS\max} - V_t = V_{GS} + v_{gs} - V_t$$

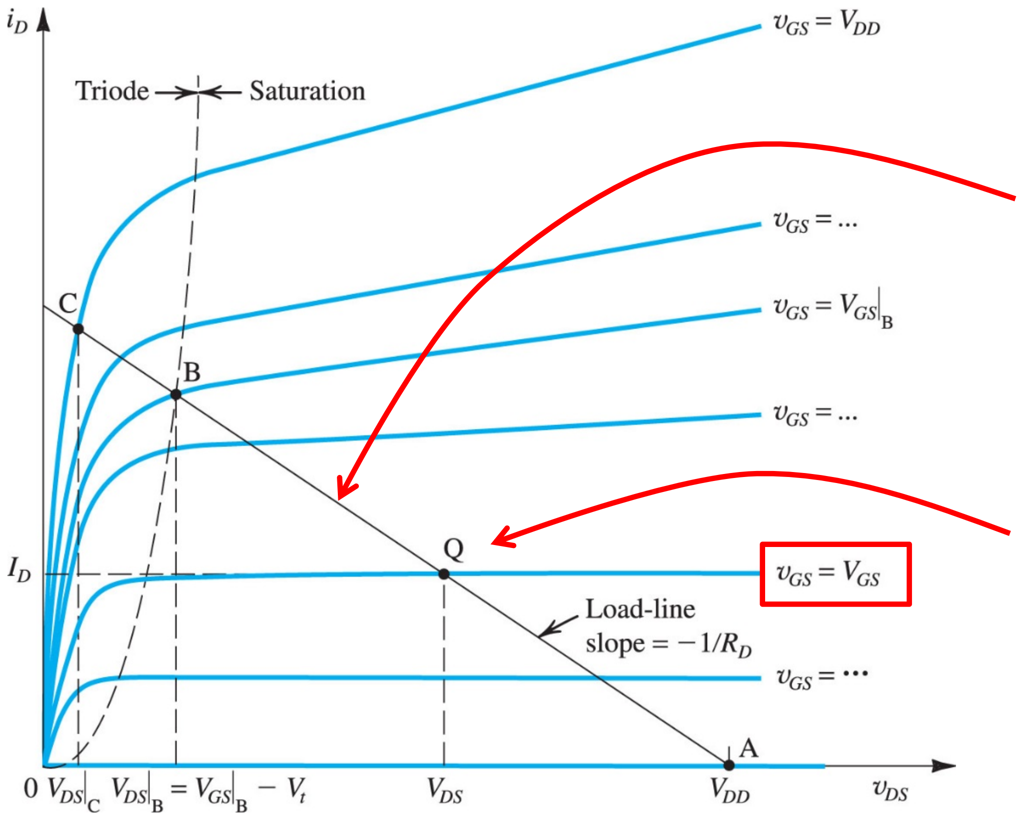
Since $v_{ds} = |A_V| v_{gs}$

$$V_{DS} - |A_V| v_{gs} \geq V_{GS} + v_{gs} - V_t$$

$$v_{gs} \leq \frac{V_{DS} - V_{OV}}{1 + |A_V|}$$

The input amplitude must be smaller than this value to ensure MOSFET stays in Saturation

Graphical Analysis with Load Line



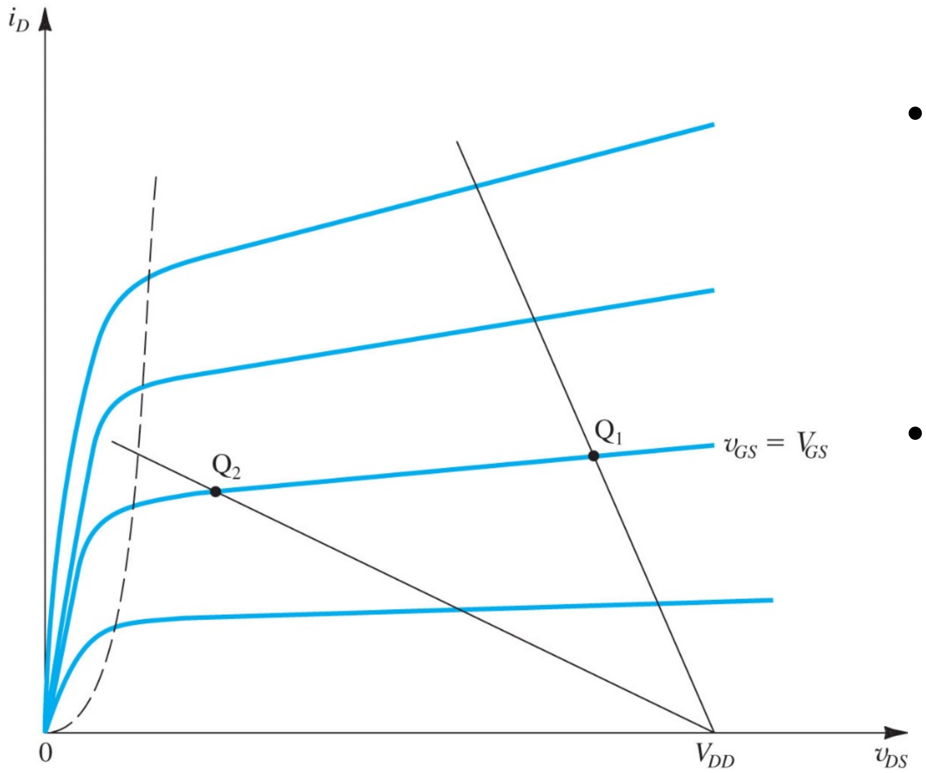
KVL:

$$V_{DD} = i_D R_D + v_{DS}$$

This is the equation for load line

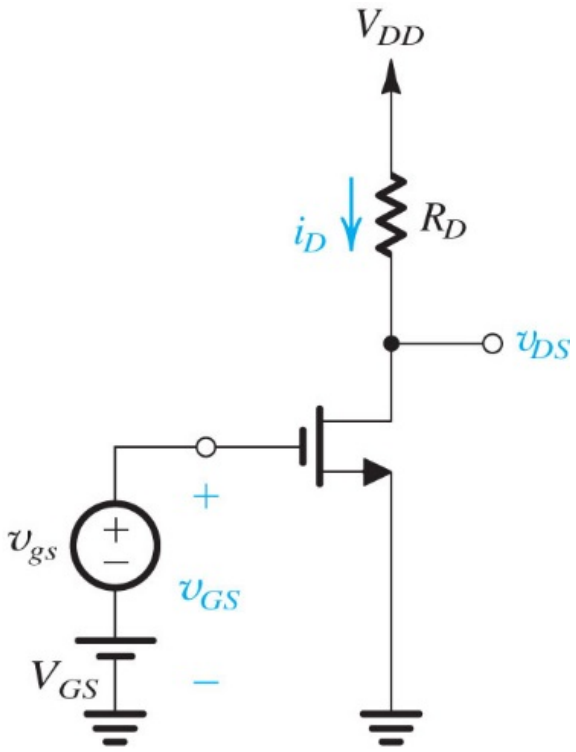
The intersection of load line and MOSFET I-V (with a specific v_{GS}) defines the bias point (Q)

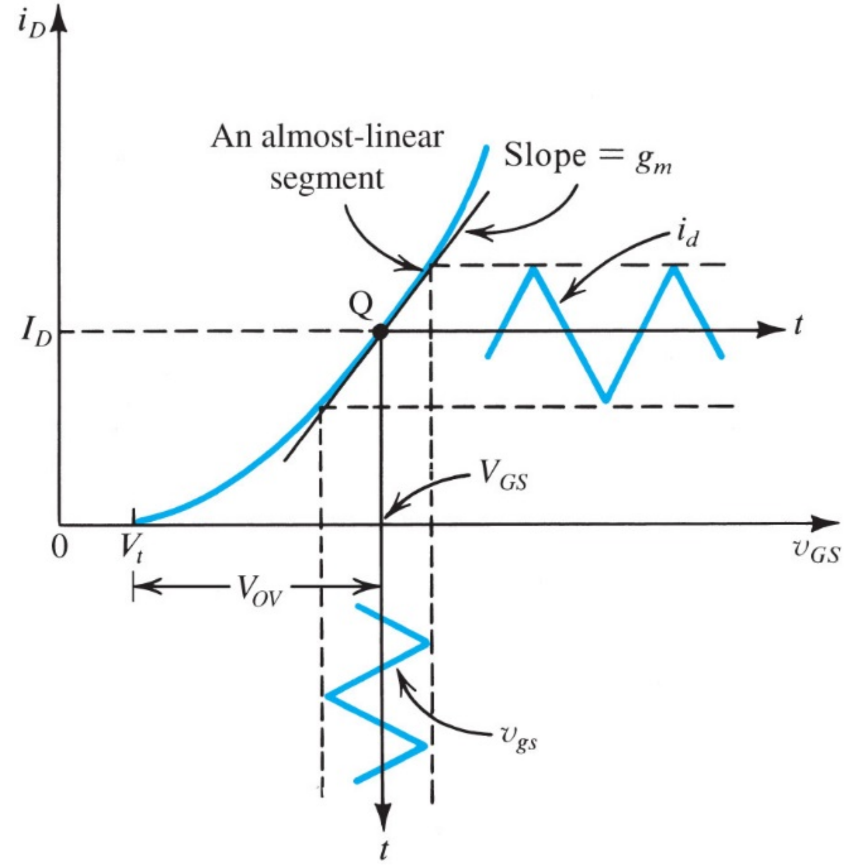
Consideration for Bias Point



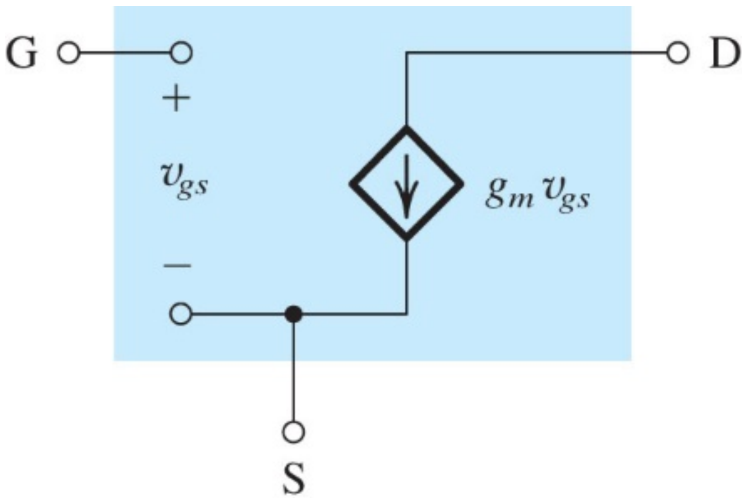
- Q_1 : too close to V_{DD}
  - Not enough room for positive signal swing
- Q_2 : too close to Triode region
  - Not enough room for negative voltage swing

MOS as Voltage-Controlled Current Source





Simplified Small-Signal Model of MOSFET



$$i_D(v_{GS}, v_{DS}) = \frac{1}{2} k_n \left(v_{GS} - V_t\right)^2 \left(1 + \lambda v_{DS}\right)$$

$$\approx \frac{1}{2} k_n \left(v_{GS} - V_t\right)^2$$

At DC bias point, Q

$$i_D = I_D + \left.\frac{\partial i_D}{\partial v_{GS}}\right|_Q v_{gs} = I_D + g_m v_{gs}$$

Transconductance:

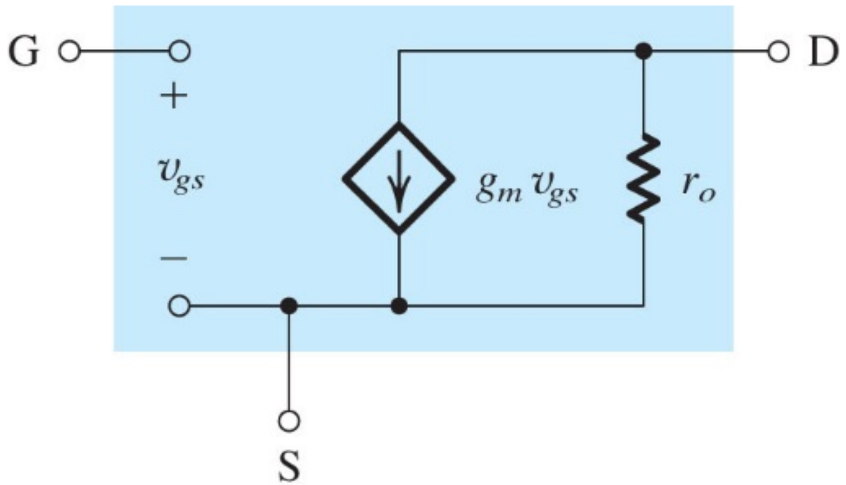
$$g_m = k_n \left(v_{GS} - V_t\right) = k_n V_{OV}$$

Alternative g_m expression:

$$g_m = \frac{2 I_D}{V_{OV}} = \sqrt{2 k_n I_D}$$

Small-Signal Model for MOSFET

Hybrid-π Model



$$i_D(v_{GS},v_{DS}) = \frac{1}{2}k_n\left(v_{GS}-V_t\right)^2\left(1+\lambda v_{DS}\right)$$

At DC bias point, Q

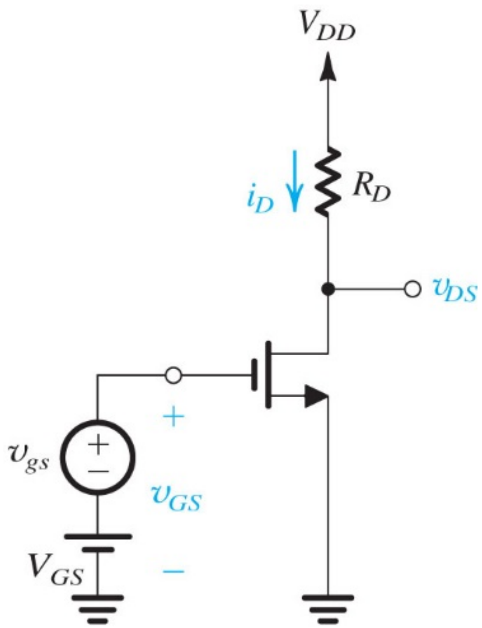
$$i_D = I_D + \left.\frac{\partial i_D}{\partial v_{GS}}\right|_Q v_{gs} + \left.\frac{\partial i_D}{\partial v_{DS}}\right|_Q v_{ds}$$

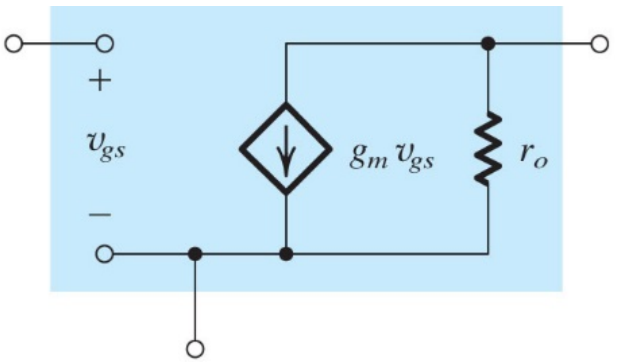
$$i_D = I_D + g_m v_{gs} + \frac{1}{r_o} v_{ds}$$

Transconductance: $g_m = k_n V_{OV}$

Output Resistance: $r_o = \frac{1}{\lambda I_D}$

Applying Small-Signal Model





Previously, we used

$$v_{DS} = V_{DD} - \frac{1}{2} k_n \left(v_{GS} - V_t \right)^2 R_D$$

$$A_V = \left. \frac{dv_{DS}}{dv_{GS}} \right|_Q = -k_n \left(v_{GS} - V_t \right) R_D$$

Voltage Gain: $A_V = -k_n V_{OV} R_D$

With small-signal model, gain can be obtained much more easily:

$$v_{ds} = -g_m v_{gs} \left(r_o \parallel R_D \right)$$

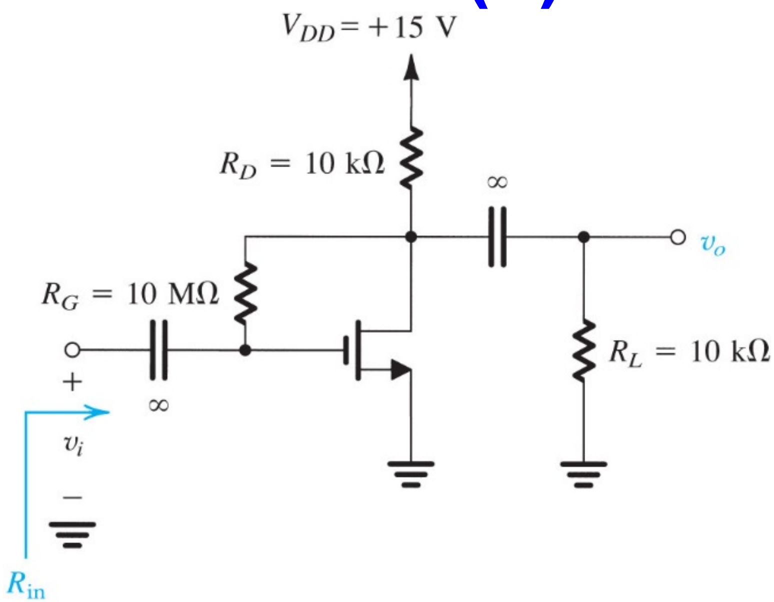
$$A_V = \frac{v_{ds}}{v_{gs}} = -g_m \left(r_o \parallel R_D \right)$$

Usually $r_o >> R_D$, $A_V \approx -g_m R_D$

Systematic Procedure for Transistor Amplifier Analysis

1. **Perform DC analysis (ignore small signal source)**
2. **Calculate small-signal parameters (g_{m}, r_o, etc)**
3. **Generate AC small-signal equivalent circuit**
   - Replace DC voltage source by short circuit
   - Replace DC current source by open circuit
   - Replace transistor by hybrid-π model
4. **Perform circuit analysis to determine voltage gain or other amplifier performance parameters**

MOSFET Amplifier Example: (1) Solve DC Bias Point



Find voltage gain for the amplifier. The MOSFET has $V_t = 1.5V$, $k_n = 0.25\ mA/V^2$ and $\lambda = (50V)^{-1}$

Coupling capacitor is open circuit in DC, and short circuit for AC signal.

To solve DC bias point, replace coupling capacitor with open circuit:

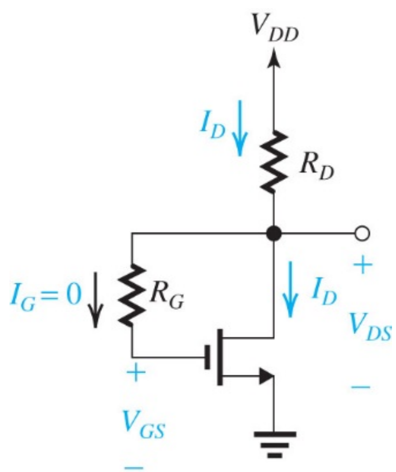
$V_{GS} = V_{DS}$ from bias connection R_G

$$V_{DD} = I_D R_D + V_{DS} = \frac{k_n}{2}\left(V_{GS} - V_t\right)^2 R_D + V_{DS}$$

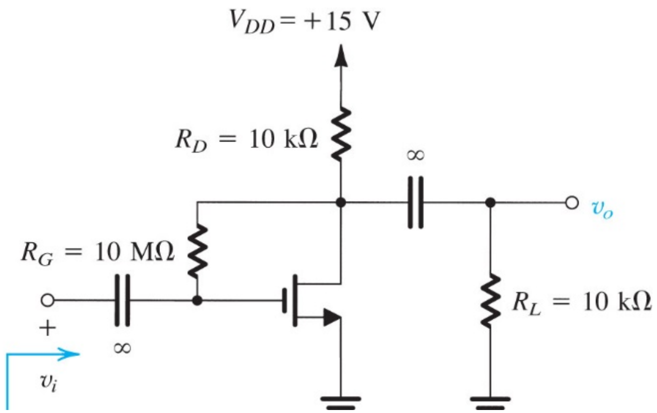
$$15 = 1.25\left(V_{DS} - 1.5\right)^2 + V_{DS}$$

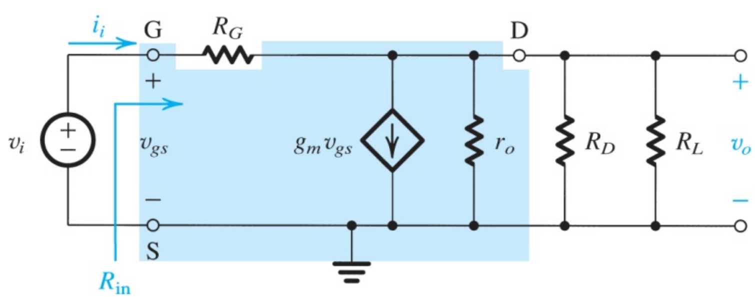
Solve quadratic equation: $V_{DS} = 4.4V$

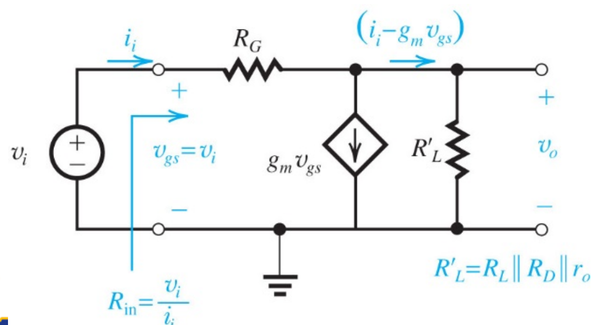
$I_D = 1.06$ mA



MOSFET Amplifier Example: (2) Solve AC Small Signal Circuit







Now replace coupling capacitor with short circuit, and replace MOSFET with hybrid-pi model with

$$g_m = k_n V_{OV} = 0.25 \cdot (4.4 - 1.5) = 0.725 \text{ mA/V}$$

$$r_0 = \frac{V_A}{I_D} = 47 \text{ k}\Omega$$

Next, simpliy resistance at output to

$$R_L^{'} = r_o \,||\, R_D \,||\, R_L = 4.5 \text{ k}\Omega$$

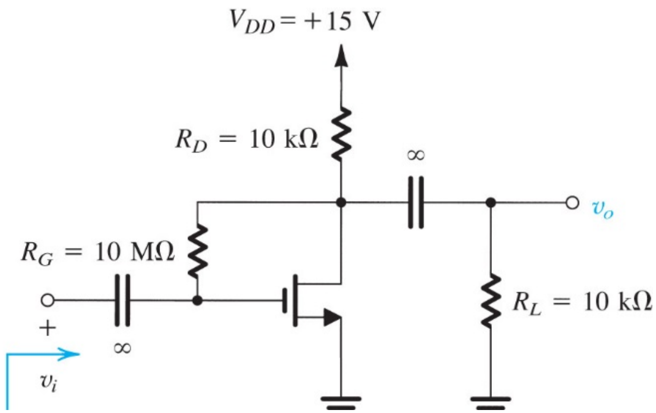
Do circuit analysis:

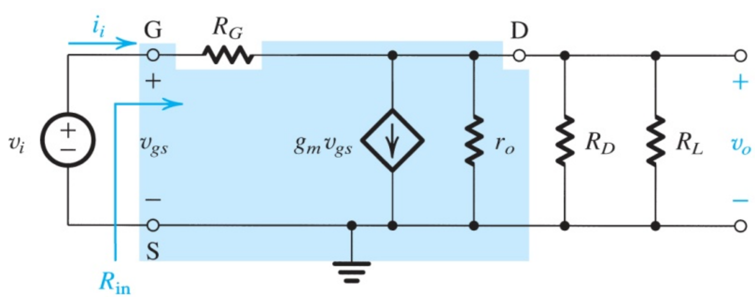
$$\begin{cases} v_o = (i_i - g_m v_{gs}) R_L^{'} \\ i_i = \dfrac{v_i - v_o}{R_G} \end{cases}$$

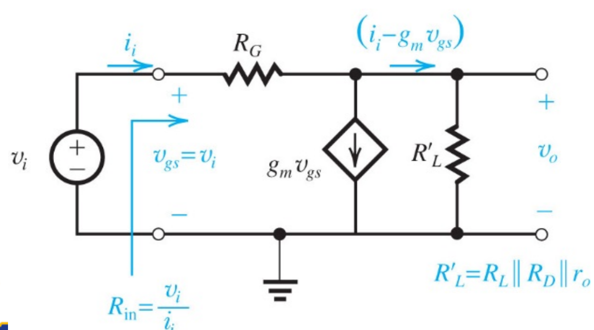
$$A_v = -g_m R_L^{'} \frac{1 - 1/g_m R_G}{1 + R_L^{'}/R_G}$$

For large R_G, $A_v \approx -g_m R_L^{'} = -0.725 * 4.5 = -3.3$ V/V

MOSFET Amplifier Example: (3) Additional Parameters of Interest







$$R_{in} = \frac{v_i}{i_i} = \frac{v_i}{\dfrac{v_i - v_o}{R_G}} = \frac{v_i}{\dfrac{v_i - A_v v_i}{R_G}} = \frac{R_G}{1 - A_v}$$

$$R_{in} = \frac{10\ \text{M}\Omega}{1 + 3.3} = 2.3\ \text{M}\Omega$$

Maximum signal that can be applied while keeping MOS in Saturation:

$$v_{DS\min} \ge v_{GS\max} - V$$

$$V_{DS} - |A_v| v_i \ge V_{GS} + v_i - V_t$$

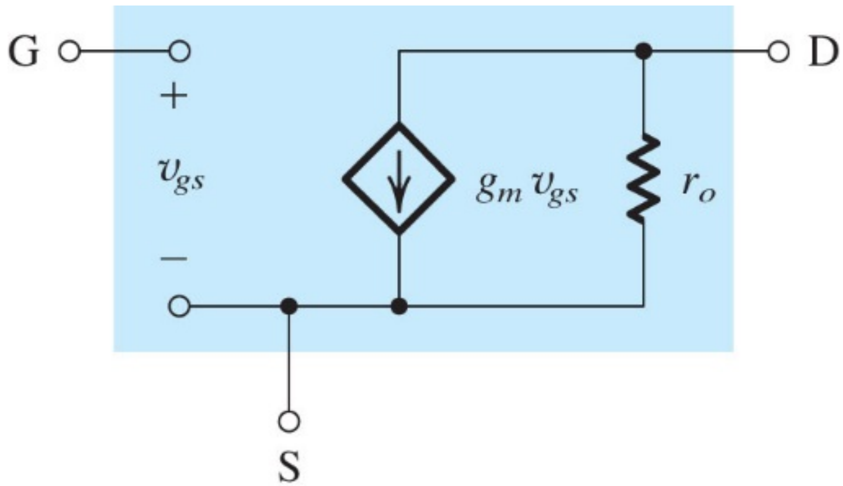
$$v_i \le \frac{V_{DS} - V_{GS} + V_t}{1 + |A_v|} = \frac{V_t}{1 + |A_v|}$$

$$\because V_{DS} = V_{GS} \text{ here.}$$

$$v_i \le 0.35\ \text{V}$$

Small-Signal Model for PMOS

Hybrid-π Model



The equivlent circuit is valid for both NMOS and PMOS.

In PMOS, use absolute sign for all parameters:
$|V_{GS}|$, $|V_t|$, $|V_{OV}|$, $|V_A|$,
and replace k_n with k_p